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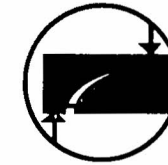
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## First International Conference on Fracture Mechanics of Concrete Structures

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# FRACTURE CHARACTERIZATION IN BLAST-LOADED CONCRETE STRUCTURES

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## ABSTRACT

The paper describes a finite element based analysis of reinforced concrete plane and axisymmetric structures under blast loading conditions. The method provides the possibility of activating different fracture mechanisms and failure modes characteristic of reinforced concrete structures. The rate sensitivity effects are also accounted for in the constitutive relations adopted for the materials in both tension and compression. The significance of history and rate dependent characterization is illustrated by means of examples.

## INTRODUCTION

An understanding of the structural behaviour under blast loading is of concern in many aspects of military and civil engineering. For more realistic analysis of two dimensional reinforced concrete structures exposed to blasts, the nonlinear finite element method offers many advantages, especially in spatial discretisation. The method lends itself to convenient modelling of material nonlinearities and different fracture mechanisms. The paper briefly describes the method, but a fuller description of the theory and implementation can be found in [1-4].

## MODELLING OF CONCRETE FRACTURE IN COMPRESSION

Concrete is idealized as a rate and history dependent elasto-viscoplastic strain hardening-softening material. The dynamic fracture of concrete in compression is modelled by generalizing the failure stress concept within the viscoplasticity theory by means of a set of variables, the values of which can be determined from empirical data. The viscoplastic behaviour is controlled by two bounding rate dependent surfaces in the stress space, the initial yield surface  $F_0$  and a failure surface  $F_f$ . The failure surface predicts failure if the state of stress satisfies the following condition

$$F_f \left( I_1, J_2, \sigma_{cd} \right) = f \left( I_1, J_2 \right) - \sigma_{cd} = 0 \quad (1)$$

where  $f \left( I_1, J_2 \right)$  is the failure function which is assumed to be a function of the first stress invariant  $I_1$  and the second deviatoric stress invariant  $J_2$ . In the above equation,  $\sigma_{cd}$  is the dynamic compressive strength of concrete and is a function of its static strength and of the effective strain rate dependent magnification factor [1]. The following function has been developed for the representation of the failure surface of concrete in the principal stress space:

$$f \left( I_1, J_2 \right) = a I_1 + \sqrt{b I_1^2 + 3 c J_2} \quad (2)$$

where  $a, b, c$  are material constants which have been evaluated elsewhere [1], using Kupfer's test results [5]. In the above formulation, the initial yield surface defines the onset of viscoplastic behaviour. When the stress state lies within this surface, concrete response is assumed to be linear elastic. Once concrete has been stressed beyond the elastic limit, a subsequent new loading surface develops. The new loading surface is assumed to have the same shape in stress space as that of the failure surface. Thus, the general form of the loading surface is proposed as

$$F_o \left( I_1, J_2, \tau \right) = f \left( I_1, J_2 \right) - \tau = 0 \quad (3)$$

in which  $\tau$  is the effective stress which is assumed to be a function of the effective viscoplastic strain  $\epsilon_{vp}$ , and the effective strain rate,  $\dot{\epsilon}_{eff}$ .

#### Concrete crushing

To model the crushing mechanism for concrete, its ultimate deformation capacity under biaxial stress state is defined as a strain controlled phenomenon. This is achieved by converting the failure function described in terms of stresses to that in terms of strains such that

$$C \left( I'_1, J'_2, \epsilon_{cu} \right) = a I'_1 + \sqrt{b I'^2_1 - 3cJ'_2} - \epsilon_{cu} = 0 \quad (4)$$

where  $I'_1$  is the first invariant of strain,  $J'_2$  is the second deviatoric strain invariant, and  $\epsilon_{cu}$  is concrete ultimate strain (0.003-0.005).

#### Degradation of concrete compressive strength

To include the plastic stiffness degradation effects in the pre- and post-failure regimes, the parameter  $\tau$  in Eqn (3) is derived as function of a history dependent plastic damage variable, effective viscoplastic strain, to fit quasi-static experimental results. Based on empirical rate sensitivity factors, this function is extended for dynamic problems. For the prefailure range, an isotropic hardening rule is developed which assumes the uniform expansion of the loading surface to be rate and history dependent. In Ref [1], the normalized effective stress is defined by the following hardening function

$$\frac{\tau}{\sigma_{cd}} = -\frac{2}{C} \left[ 1 - C + \frac{2C}{\epsilon_{cd}} \epsilon_{vp} \right]^{1/2} - \frac{2}{C} \left[ 1 - C + \frac{C}{\epsilon_{cd}} \epsilon_{vp} \right] \quad (5)$$

in which  $\epsilon_{cd}$  is the dynamic peak compressive strain and  $C$  is a constant derived as a function of concrete elastic limit. In the postfailure range, the loading surface shrinks with the increase in viscoplastic strain. A rate dependent softening rule has been defined [1] in which the effective stress is assumed as a function of the dynamic compressive strength and the postfailure viscoplastic energy,  $\kappa$ , such that

$$\tau = \sigma_{cd} e^{-\beta\kappa} \quad (6)$$

in which  $\beta$  is a concrete softening constant. Excellent agreement [1] has been obtained with Kupfer's results [5] for both the hardening and softening functions. To evaluate the viscoplastic strain rate vector,  $\dot{\epsilon}_{vp}$ , the classic flow rule is modified to include rate dependence of inelastic deformations. The evolution law for the internal variable of associated rule, the fluidity parameter  $\gamma$ , is derived as a semi-empirical function of the effective strain rate.

$$\dot{\epsilon}_{vp} = \gamma \left( \dot{\epsilon}_{eff} \right) \langle \varnothing (F_o) \rangle a \quad (7)$$

where  $\varnothing (F_o)$  is the flow function and  $a$  is the flow vector. From the governing uniaxial elasto-viscoplastic stress strain relation [1], and the curve fitting of experimental results, the fluidity parameter is found as a function of the effective strain rate and the concrete compressive strength.

#### IDEALIZATION OF CONCRETE TENSILE FRACTURE

Tensile fracture is modelled here by treating concrete as a linear elastic strain softening material [1, 4] with smeared fixed crack formulation.

##### Crack initiation criterion

The onset of tensile cracking is assumed to be governed by a rate sensitive strain criterion. The influence of deformation velocity is considered by raising the cracking strain to relative the static condition using a rate dependency factor.

$$\epsilon_{cr} \geq \epsilon_{td} \quad (8)$$

where  $\epsilon_{td}$  is the dynamic cracking strain of concrete which is defined [1] as a function of the corresponding static strain and an empirical effective strain rate dependent magnification factor.

##### Degradation of tensile strength of cracked concrete

Upon cracking the gradual loss of tensile strength of concrete is governed by a nonlinear reversible tension softening rule based on the concrete fracture energy  $G_f$ , the crack characteristic length  $L_c$ , the static tensile strength  $\sigma_{ts}$  and static cracking strain  $\epsilon_{ts}$

$$\sigma = \sigma_{ts} e^{-\frac{\epsilon - \epsilon_{ts}}{\psi}}, \quad \psi = \left( G_f - 0.5 \sigma_{ts} \epsilon_{ts} L_c \right) / \sigma_{ts} L_c \quad (9)$$

The fracture energy concept leads to a nonlocal format of the equivalent softening relation necessary to meet objectivity of analysis with respect to mesh elements size.

#### CHARACTERIZATION OF CONCRETE FRACTURE IN SHEAR

##### Degradation of shear strength of cracked concrete

Considerable shear can be transferred across cracks due to aggregate interlock and dowel action of steel bars. In the smeared crack approach, the shear modulus of cracked concrete is obtained by using a reduction factor, which depends on the tensile strain  $\epsilon$  across the crack.

$$\beta_c = 1 - \frac{\epsilon_t}{0.004}; \quad \beta = 0 \text{ if } \epsilon_t \geq 0.004 \quad (10)$$

##### Detection of fracture localization and diagonal shear cracks

Under blast conditions, sudden and extensive diagonal cracking occurs within a few time steps. Some recommendations on computational strategies to be adopted are now given, based on numerical results [1]. First, displaying only full cracks, i.e. cracks for which the normal strain is beyond the ultimate strain of the tensile softening branch, is necessary to reveal any fracture localization in the structure behaviour as well as to uncover the shear-type cracks in the predicted crack patterns. Second, the use of Gaussian quadrature with a reduced 2x2 order for the integration of stress fields proves to be a successful means of avoiding stress-locking phenomena associated with strain localization mechanism [1]. This results in a partial release of the continuity requirements imposed by shape functions assumed for the finite elements.

#### MODELLING OF YIELDING OF STEEL REINFORCEMENT

Steel is modelled as a uniaxial strain rate dependent elasto-viscoplastic material in tension and compression in which the yield stress  $\sigma_{yd}$  is rate dependent. Beyond yield, the effective stress level is governed by a linear strain hardening function such that

$$\tau = \sigma_{yd} + H \epsilon_{vp} \quad (11)$$

where H is the hardening modulus of steel. To calculate the viscoplastic strain rate, the associated flow rule is modified in a similar fashion to concrete where the fluidity parameter is derived as a semi-empirical function of the strain rate [1].

#### FINITE ELEMENT IMPLEMENTATION AND NUMERICAL APPLICATIONS

For the spatial discretisation of the nonlinear dynamic equilibrium equations, 8-node isoparametric elements have been employed for concrete with embedded bars to simulate reinforcement. Perfect bond is assumed

between steel and concrete. Explicit central difference scheme has been used for temporal discretisation of dynamic equations while explicit Euler integration has been adopted for time rate constitutive relations [1, 4]. To implement the proposed fracture models, a versatile computer program, FEABRS, has been developed for linear and nonlinear dynamic analysis of two dimensional concrete structures. The program has been used to study the response characteristics of several structures under impulsive and blast loading [1-4]. In Figs 1-4, comparisons are made with test results of Ref [6] where many aspects of concrete fracture and the generality of the proposed models are well demonstrated by the analysis.

### CONCLUSION

For the finite element analysis of concrete structures, a numerical model is presented for characterization of fracture mechanism under blast loading. The proposed rate and history elasto-viscoplastic constitutive model which includes strength degradation effects in the pre- and post-failure regimes, is suitable for modelling the concrete dynamic fracture in compression. For idealization of concrete tensile fracture, the adopted rate sensitive cracking condition in conjunction with fracture energy dependent tension softening rule proved to be a powerful numerical tool. In addition to a fracture strain based-shear retention factor, some recommendations are described for detecting diagonal shear cracks and fracture localization mechanisms. Steel yielding is considered by a modified viscoplasticity theory. An illustrative example is included.

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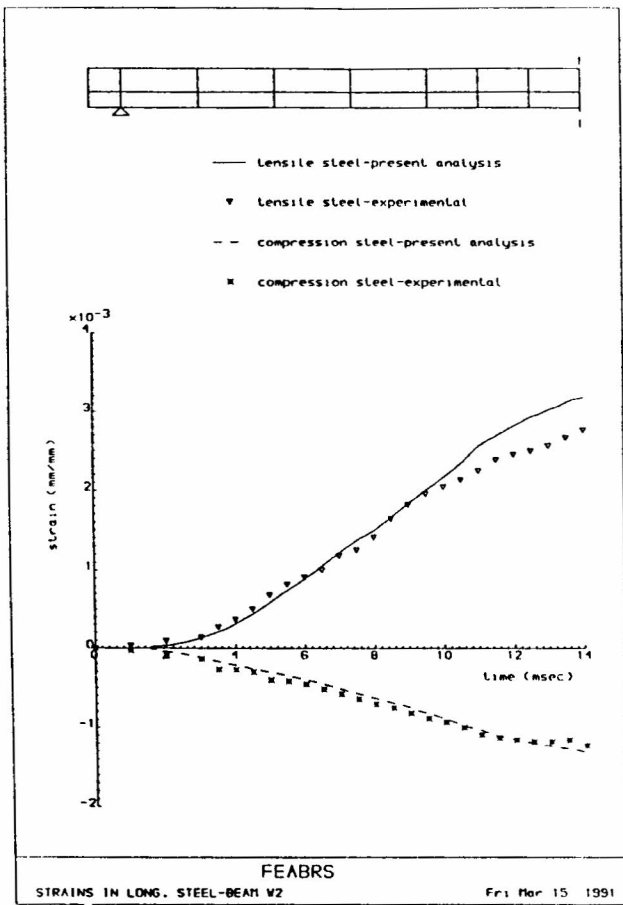


Fig 1: Strain history of steel

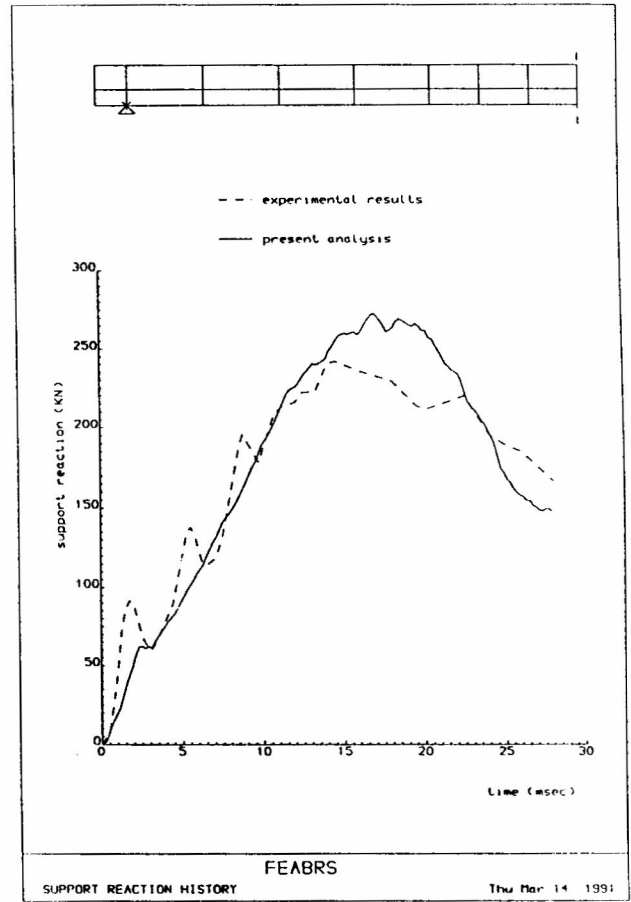


Fig 2: End reaction history

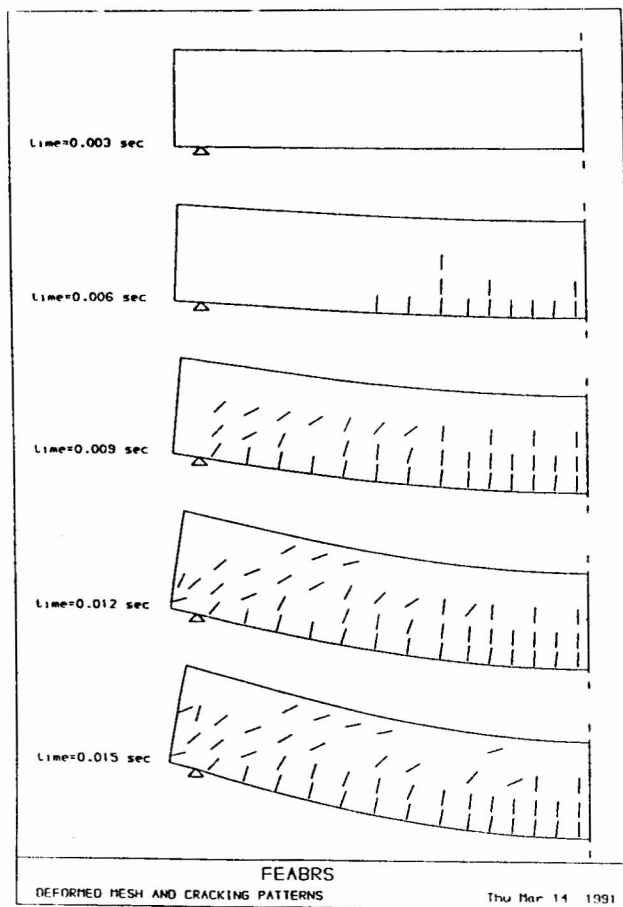


Fig 3: Cracking patterns

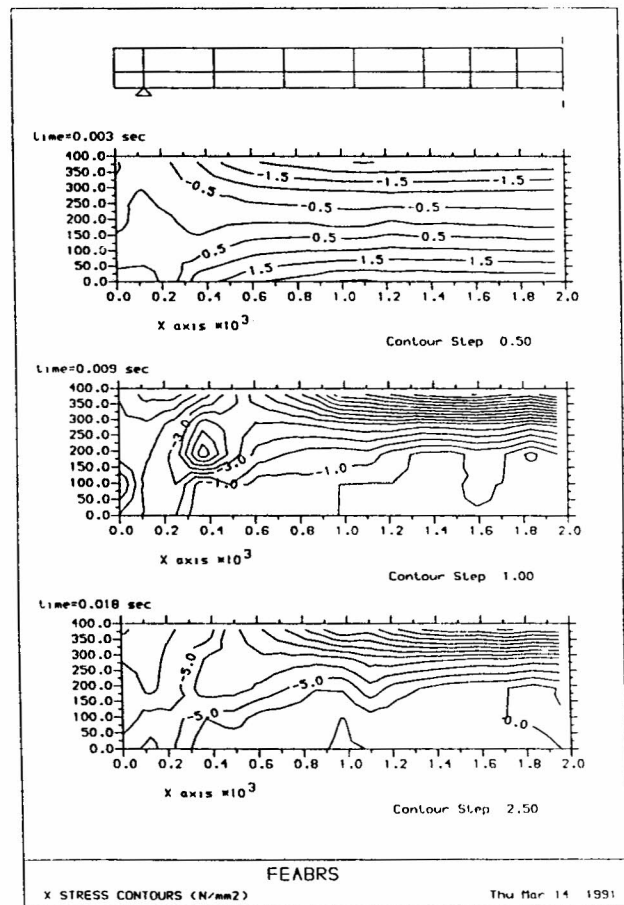


Fig 4: Longitudinal stress